On particle production by classical backgrounds

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Abstract

The theory of quantum fields in classical backgrounds is re-examined for the cases in which the Lagrangian is quadratic in quantum fields. Various methods that describe particle production are discussed. It is found that all methods suffer from certain ambiguities, related to the choice of coordinates, gauge, or counter-terms. They also seem to be inconsistent with the conservation of energy. This suggests that such classical backgrounds may not cause particle production.

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One of the main differences between the first quantization and the second quantization (quantum field theory) is the fact that the latter is able to describe production and destruction of particles. A particularly interesting, but still experimentally unverified prediction of quantum field theory is that the vacuum may be unstable, i.e. that a nontrivial classical background may cause particle production. For example, such an effect is believed to exist in the electromagnetic [1] and gravitational [2] background. However, the concept of particle is not a fundamental concept in quantum field theory. It is well defined only for free fields, i.e. fields described by a free Lagrangian. On the other hand, production and destruction of particles may occur only if an interaction is present. Therefore, it is convenient to write the Lagrangian as $\mathcal{L} = \mathcal{L}_{\rm free} + \mathcal{L}_{\rm int}$ and use the interaction picture. In the general case, it is not clear how to separate the "free" part from the "interaction" part in a unique way. As we demostrate below, this is not merely a matter of convenience in some cases. Instead, the physical predictions related to particle production may depend on what we mean by "free" particles.

In this letter we study systems described by Lagrangians quadratic in a quantum field ϕ. Such Lagrangians can be written in an elegant way as

$$\mathcal{L} = \phi^\dagger J \phi \; . \tag{1}$$

Here J can be a c-number function or a derivative operator, but not a quantum-field operator. To clarify the meaning of J, we give a few examples. If ϕ is a free scalar complex field, then $J = \overleftarrow{\partial^\mu}\partial_\mu - m^2$. (The arrow denotes that the operator acts on the left.) The generalization of this for a curved background described by the metric $\bar{g}_{\mu\nu}$ is $J = \overleftarrow{\partial^\mu}|\bar{g}|^{1/2}\partial_\mu - m^2$. If ϕ is a Dirac spinor interacting with the electromagnetic background $\bar{A}_\mu$, then $J = \gamma^0[i\gamma^\mu(\partial_\mu + ie\bar{A}_\mu) - m]$.

The quantity J can always be written as

$$J = J_{\text{free}} + J_{\text{int}} \ , \tag{2}$$

which defines the interaction picture and the corresponding Feynman rules. The elementary vertex takes the form as in Fig. 1. There are three possible physical interpretations of the diagram in Fig. 1. The first interpretation is the renormalization of the free propagator. The second interpretation is scattering caused by the classical background J_{int}. The third interpretation is pair production induced by the classical background J_{int}. We argue below that the third interpretation may not be physical. Irrespective of the interpretation, the most general diagram can be represented as a combination of disconnected diagrams with two external legs. The diagram with two external legs calculated at all orders of perturbation theory is represented by Fig. 2. It is important to note that, for a given J, the separation (2) is not unique. Let us discuss a few examples.

We begin with the case $J_{\text{free}} = \overleftarrow{\partial^\mu}\partial_\mu$, $J_{\text{int}} = -m^2$. In this case, the most natural interpretation of Figs. 1 and 2 is the renormalization of the propagator. Indeed, the diagrams of Fig. 2 can be easily summed [3], leading to the usual massive propagator represented by the left-hand side of Fig. 2. The scattering interpretation is also meaningful. If we define scattering as a deviation of the particle trajectory from the trajectory that would be realized if the particle were "free", then it becomes clear that the mass term can be interpreted in such a way, simply because the trajectory of a massive particle is different from that of a massless particle. Pair production is also possible, in the sense that the amplitude for the production of a particle-antiparticle pair does not vanish. However, real massless particles cannot be produced simply because it is forbidden *kinematically*. Such a production would not be consistent with the conservation of energy.

Let us now discuss a more interesting case, in which $J_{\text{free}} = \overleftarrow{\partial^\mu}\partial_\mu - m^2$ (for bosons) or $J_{\text{free}} = \gamma^0(i\gamma^\mu\partial_\mu - m)$ (for fermions). This corresponds to the usual definition of the "free" Lagrangian. We assume that J_{int} is a non-trivial function, corresponding to a gravitational, electromagnetic, or any other background field. In this case, a natural interpretation of Figs. 1 and 2 is the scattering interpretation. The propagator-renormalization interpretation is also possible. Instead of summing the diagrams of Fig. 2, the renormalized propagator can be found more directly in the following way: (We present the method for bosons, while that for fermions is essentially the same.) The Lagrangian (1) provides that the equation of motion for ϕ is a linear homogeneous equation, so the solution can be expanded as

$$\phi(x) = \sum_k a_k f_k(x) + b_k^\dagger g_k^*(x) \ . \tag{3}$$

The operators a_k, $a_k^\dagger$, b_k and $b_k^\dagger$ satisfy the usual algebra of raising and lowering operators.

This allows us to introduce the state $|0\rangle$ with the property $a_k|0\rangle = b_k|0\rangle = 0$. The propagator $G(x,y)$ is then given by

$$G(x,y) = \langle 0|O\phi^{\dagger}(x)\phi(y)|0\rangle \,, \tag{4}$$

where O is an ordering operator, depending on which propagator we want to obtain. For example, the Feynman propagator corresponds to the time ordering.

The crucial question is whether the diagrams in Figs. 1 and 2 may correspond to pair production in the case of non-trivial $J_{\rm int}$. The corresponding amplitude does not vanish. However, the process is kinematically forbidden again, because the production of particles increases the energy of the system. Nevertheless, the usual argument that supports the pair production is a claim that the pair production causes a backreaction on the "source" $J_{\rm int}$, such that the energy is conserved. To avoid a possible misunderstanding, note that this backreaction cannot correspond to a recoil of a massive object that generates $J_{\rm int}$, because the recoil only saves the conservation of the 3-momentum, not the conservation of energy. Therefore, the physical mechanism that causes this hypothetical backreaction is not clear. The backreaction is not a consequence of the basic principles of quantum field theory. If one insists on retaining the possibility of pair production, then the backreaction should be included in a vague way, by hand. If the backreaction really exists, then it is not clear why it does not exist for the case $J_{\rm int} = -m^2$ discussed above. For example, particles could be spontaneously created in flat space-time, which would be accompained by a spontaneous change of the background metric, such that the negative gravitational energy would cancel the positive energy of the particles created. Of course, the experimental observation that particles are not spontaneously created in flat space-time is not a surprise. Our point is that, for any $J_{\rm int}$, the existence of the backreaction does not seem to be reasonable. Therefore, the conclusion that $J_{\rm int}$ does not cause pair production seems to be the most reasonable.

There are also other arguments against pair production, which do not rely on energy conservation. Let us pay particular attention to the case of electromagnetic background. In this case, the full Lagrangian $\mathcal{L}$ is gauge invariant, but its separate parts $\mathcal{L}_{\rm free}$ and $\mathcal{L}_{\rm int}$ are not. Therefore, the physical results that follow from the calculation of the diagrams in Figs. 1 and 2 depend on the choice of gauge. Of course, the renormalized propagator (4) should depend on gauge. However, the number of produced particles should not depend on gauge. If the calculation is based on the diagrams in Figs. 1 and 2, then the only gauge-invariant conclusion related to pair production is that these diagrams do not describe pair production.

The particle production by an electromagnetic background can be studied in a gauge-invariant way, by calculating the effective action $W = \int d^4x \mathcal{L}_{\rm eff}$. It is represented by one-loop diagrams that can be obtained by gluing together the external legs of the diagrams in Fig. 2. The calculation can be performed by using Schwinger's method [1], which is often viewed as the clearest proof that a constant background electric field causes particle production. Since $\langle 0 \text{ out}|0 \text{ in}\rangle = \exp(iW)$, the quantity

$$|e^{iW}|^2 = e^{-2{\rm Im}W} \tag{5}$$

represents the probability that no actual pair creation occurs. Schwinger's method leads to an exact expression for $\mathcal{L}_{\rm eff}$ for the case when the background field $\bar{F}_{\mu\nu}$ is constant.

For this case, his method gives that $\mathrm{Im}\mathcal{L}_{\rm eff} = 0$ when $\bar{\mathbf{E}} \cdot \bar{\mathbf{B}} = 0$ and $\bar{\mathbf{E}}^2 - \bar{\mathbf{B}}^2 < 0$, whereas $\mathrm{Im}\mathcal{L}_{\rm eff} > 0$ when $\bar{\mathbf{E}} \cdot \bar{\mathbf{B}} = 0$ and $\bar{\mathbf{E}}^2 - \bar{\mathbf{B}}^2 > 0$. The latter case corresponds to an unstable vacuum, i.e. to particle creation. However, there are several weaknesses of the method presented in [1]. First, the method expresses $\mathcal{L}_{\rm eff}$ as a certain integral over a real variable s. This $\mathcal{L}_{\rm eff}$ is divergent and *real*. Of course, the divergence is expected because it is a one-loop calculation. In this method, the divergences are related to the poles of the subintegral function. Schwinger obtained the imaginary contributions in an artificial way, by replacing the contour of integration with a contour that avoids the poles. Second, a pole exists even for the case $\bar{F}_{\mu\nu} = 0$. Schwinger removed this undesirable term by hand, by introducing an appropriate counter-term. If the addition of such counter-terms is allowed, then one can also introduce a counter-term that will cancel the contribution to $\mathrm{Im}\mathcal{L}_{\rm eff}$ that comes from finite $\bar{F}_{\mu\nu}$. Such a renormalization seems to be the most natural and leads to a stable vacuum. Third, if $\bar{F}_{\mu\nu}(x)$ is not constant, such that the Fourier transform $\bar{F}_{\mu\nu}(k)$ possesses contributions from $k^2 > 4m^2$, then $\mathrm{Im}W$ can be *negative* (at least at the lowest order of perturbation theory). Schwinger derived it (see (6.33) in [1]), but did not comment it. This is a pathological result because it leads to the result that the probability that no actual pair creation occurs can be larger than 1. This pathology suggests again that the contour of integration should remain on the real axis or that the counter-terms should be introduced for all imaginary contributions. It is not consistent to introduce the counter-terms only for negative contributions to $\mathrm{Im}W$.

At this point, it is instructive to discuss other methods that predict particle production by a background source $J_{\rm int}$. The most popular method is the Bogoliubov transformation. It is important to note that this method is applicable only to systems that can be described by Lagrangians of the form (1) and is equivalent to the squeezed-state method [4]. In this method, the definition of particles is based on the identification of positive-frequency solutions to the equation of motion. The particle production described in this way is also inconsistent with energy conservation [5]. Another problem with the Bogoliubov-transformation method is the fact that the identification of the positive-frequency solutions to the classical equations of motion depends on the choice of coordinates; different choices of the time coordinate lead to different particle contents of the same quantum state [2]. It is often argued that this does not make the theory inconsistent, because different observers have different natural choices of the time coordinate, so different observers observe different particle contents. However, in the case of an electromagnetic background, the problem becomes even more serious, because the identification of the positive-frequency solutions depends also on the choice of gauge [6]. In particular, for the case $\bar{F}_{\mu\nu} = 0$, the "natural" gauge is $\bar{A}_{\mu} = 0$, which defines the "natural" vacuum. By using a different gauge one can obtain that this vacuum is a many-particle state. An appropriate gauge leads to a thermal distribution of particles in the "natural" vacuum, leading to an electromagnetic analog of the Unruh effect. On the other hand, physical quantities should not depend on gauge. All this suggests that the Bogoliubov transformation is not a well-founded method for studying particle creation.

Finally, let us note that particle production by an electromagnetic background can be viewed as a tunelling process [7, 8]. In general, this method is also gauge dependent.

Before drawing a conclusion, let us shortly discuss a more general case, in which J in (1) can be a quantum-field operator. Our discussion above does not refer to such a case. The most interesting example is the spinor quantum electrodynamics, in which the full quantum-field operator $\hat{A}_\mu$ can be written as

$$\hat{A}_\mu = \bar{A}_\mu + A_\mu \ , \tag{6}$$

where $\bar{A}_\mu$ is a c-number background and A_μ is an operator with the property $\langle 0|A_\mu|0\rangle = 0$. Such an interaction allows physical processes that are forbidden when the electromagnetic background described by $\bar{A}_\mu$ is not present. For example, the pair production $\gamma \rightarrow e^+e^-$ becomes kinematically allowed because the massive object that generates $\bar{A}_\mu$ can recoil. Such a process has been measured [9] and is not in contradiction with our theoretical results that suggest that the process vacuum $\rightarrow e^+e^-$ is forbiden. The latter process has never been measured.

As we have seen, all methods that predict unstable vacuum in systems described by a Lagrangian of the form (1) suffer from certain ambiguities related to the choice of coordinates, gauge, or counter-terms. Besides, all methods have problems with the conservation of energy. This suggests that we should define particles in a general covariant and gauge-invariant way and that this definition should automatically imply that the number of particles is conserved for any system described by a Lagrangian of the form (1), where J is not a quantum-field operator. Since it is usual to define the concept of particles by the "free" Lagrangian that conserves the number of particles, our discussion suggests that any Lagrangian of the form (1) (where J is not a quantum-field operator) should be viewed as a "free" Lagrangian. In particular, this will provide the general covariance and gauge invariance of the concept of particles because (1) is invariant with respect to general coordinate transformations and gauge transformations. If the separation of (1) into the "free" and "interacting" part is made in the usual way, then the separate parts are not general invariant and gauge invariant. Note also that a gauge transformation $\hat{A}_\mu \rightarrow \hat{A}_\mu + \partial_\mu\lambda$, where λ is a c-number function, can always be written as

$$\begin{aligned} \bar{A}_\mu &\rightarrow \bar{A}_\mu + \partial_\mu\lambda \ , \\ A_\mu &\rightarrow A_\mu \ , \end{aligned} \tag{7}$$

which means that the interaction Lagrangian $\mathcal{L}_{\rm int} = -eA_\mu\bar{\psi}\gamma^\mu\psi$ is gauge invariant with respect to such gauge transformations. In addition, our discussion suggests that one has to reject a common belief that the definition of particles should be closely related to the definition of positive frequencies. The explicit definition of particles that obeys all properties suggested in this paragraph will be given elsewhere.

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References

[1] J. Schwinger, Phys. Rev. 82 (1951) 664.

[2] N. D. Birrell and P. C. W. Davies, Quantum fields in curved space (Cambridge Press, NY, 1982).

[3] L. H. Ryder, Quantum Field Theory (Cambridge University Press, Cambridge, 1984).

[4] L. P. Grishchuk and Y. V. Sidorov, Phys. Rev. D 42 (1990) 3413.

[5] H. Nikolić, hep-th/0103053.

[6] T. Padmanabhan, Phys. Rev. Lett. 64 (1990) 2471.

[7] R. Brout, R. Parentani and Ph. Spindel, Nucl. Phys. B 353 (1991) 209.

[8] K. Srinivasan and T. Padmanabhan, Phys. Rev. D 60 (1999) 024007.

[9] L. E. Wright, K. K. Sud and D. W. Kosik, Phys. Rev. C 36 (1987) 562.

Figure 1: The elementary vertex related to the interaction of the form $\mathcal{L}_{\rm int} = \phi^\dagger J_{\rm int}\phi$, where $J_{\rm int}$ is not a quantum-field operator.

Figure 2: The diagram with two external legs calculated at all orders of perturbation theory for the interaction as in Fig. 1.

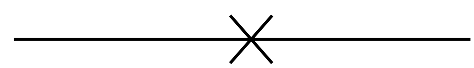

Fig. 1

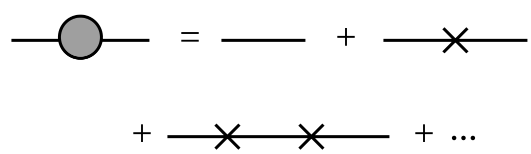

Fig. 2